Mid-term exam of Optical Quantum Electronics

October 5, 2018

Exam rules

1. The exam is open book (with internet access). Please cite the reference clearly in your answer sheets. (e.g.: Wikipedia, quantum mechanics page; book: nonlinear optics, Boyd, chapter 2.)

2. It is not allowed to discuss the mid term problems with any person until the class on Oct 15th.

3. Exam time will be 4 hours. Write down your start and stop time and date on the first page. You are allowed to take one break during the exam. The break can be arbitrarily long.

4. All students shall submit their answers to the instructor in class on Oct 15th (Monday).

5. NO quantum mechanics is needed for the exam.

6. For students who are auditing this class: you can use the score of this exam when you enroll in the same course in the future. Your score is valid as long as I'm the instructor of optical quantum electronics. Keep a copy of your graded exam sheet in case I lose your score.

Problems

Silicon is the most popular semiconductor material in the electronic industry. The fabrication technology of silicon is far more mature and affordable than any other semiconductor materials. Therefore, it is interesting to consider if silicon can be a good material for integrated photonics device. Here, let's examine if silicon is a good material for second harmonic generation (optical frequency doubling).

Second harmonic generation is a type of nonlinear phenomenon where part of the input light with frequency ω can be converted into light with frequency 2ω. The efficiency of this wavelength conversion is critical. The nonlinear efficiency can be defined as the power of 2ω light at the output port versus the power of ω light at the input port. Assuming the waveguide is along the z-direction, we have nonlinear efficiency:

$$\Gamma = P_2(z = L)/P_1(z = 0). \tag{1}$$

The electricalf field in the material can be expressed as:

$$E(z,t) = E_1(z)e^{-i\omega t+ik_1 z} + E_2(z)e^{-2i\omega t+ik_2 z} + c.c. \tag{2}$$

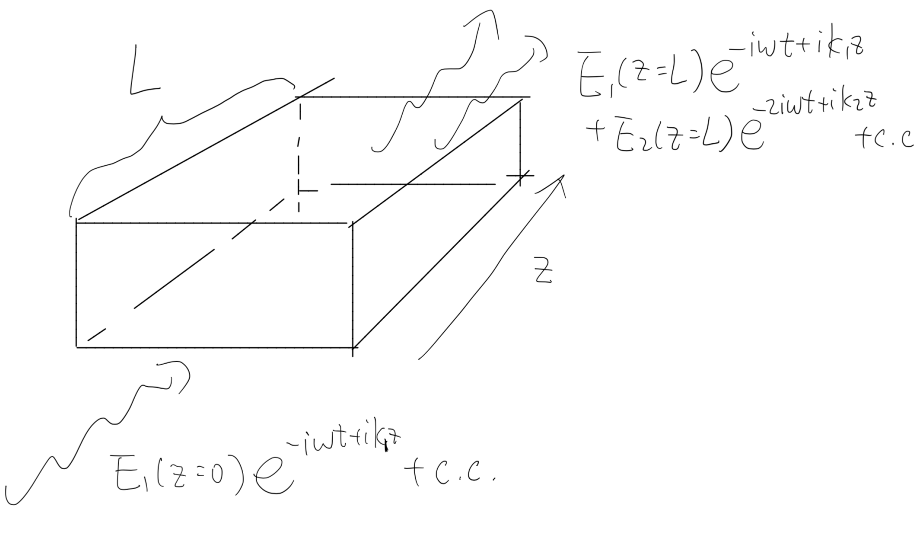
A real-world application of second harmonic generation is the green laser pointer. It is relatively challenging to make laser at 532 nm directly. However, the laser technology at 1064 nm wavelength is well developed. So it makes sense to use second-harmonic generation to create 532 nm laser light from a 1064 nm laser source.

Question 1:

Write down the wave-vector k in terms of wavelength λ and refractive index n.

Question 2:

What is the frequency of 1064 nm and 532 nm? Accurate to the third digit.



Question 3:

Which frequency in Question 2 corresponds to frequency $\omega/2\pi$? Here, ω is defined in the electrical field equation:
$E(z,t) = E_1(z)e^{-i\omega t + ik_1 z} + E_2(z)e^{-2i\omega t + ik_2 z} + c.c.$.

Question 4:

Search online: what is the refractive index of silicon at 1064 nm and 532 nm? Accurate to the second digit. If you happen to find both real and imaginary part of the refractive index, only write down the real part here.

Question 5:

What is the numerical value of k_1 and k_2?

Additional information:
In our class, we have studied the Maxwell equation and we derived a nonlinear equation:

$$\nabla^2 E(z,t) - \mu_0 \epsilon_0 \epsilon_r \frac{\partial^2 E(z,t)}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}(z,t)}{\partial t^2}, \tag{3}$$

where P_{NL} is the nonlinear polarization. In the case of second harmonic generation, it can be expressed as $P_{NL}(z,t) = \epsilon_0 \chi_2 E^2(z,t)$.

Question 6:

Write down $P_{NL}(z,t)$ in terms of $E_1(z)$, $E_2(z)$, ω, k_1, k_2, z and t. DO NOT plug in the numerical value from question 2, 3, or 5.

Additional information:
Let's work in the low efficiency regime first, where we can assume $E_1 \gg E_2$ for all z from 0 to L.

Question 7:

What are the leading terms in $P_{NL}(z,t)$? (Hint: throw away all terms with $E_2(z)$.)

Question 8:

Now use the answer of Question 7: what is $\mu_0 \partial^2 P_{NL}(z,t)/\partial t^2$?

Additional information:
Now let's work on the left-hand side of nonlinear equation (3). From the class, we know that by using slow varying amplitude approximation, we can arrive at:

$$\begin{aligned}\nabla^2 E(z,t) - \mu_0 \epsilon_0 \epsilon_r \frac{\partial^2 E(z,t)}{\partial t^2} &= -k_1^2 E_1(z)e^{-i\omega t + ik_1 z} - k_2^2 E_2(z)e^{-2i\omega t + ik_2 z} \\ &+ 2ik_1 e^{-i\omega t + ik_1 z}\frac{\partial}{\partial z}E_1(z) + 2ik_2 e^{-2i\omega t + ik_2 z}\frac{\partial}{\partial z}E_2(z) \\ &+ \mu_0 \epsilon_0 \epsilon_{r,1}\omega^2 E_1(z)e^{-i\omega t + ik_1 z} + 4\mu_0 \epsilon_0 \epsilon_{r,2}\omega^2 E_2(z)e^{-2i\omega t + ik_2 z} + c.c.\end{aligned} \tag{4}$$

Question 9:

Given $\mu_0\epsilon_0 = 1/c^2$ (c is the speed of light) and $\epsilon_{r,1} = n_1^2$ and $\epsilon_{r,2} = n_2^2$, show that $k_1^2 = \mu_0\epsilon_0\epsilon_{r,1}\omega^2$ and $k_2^2 = 4\mu_0\epsilon_0\epsilon_{r,2}\omega^2$.

Additional information:
With question 9, we can simplify the left-hand side of equation (3) to:

$$\nabla^2 E(z,t) - \mu_0\epsilon_0\epsilon_r \frac{\partial^2 E(z,t)}{\partial t^2} = 2ik_1 e^{-i\omega t + ik_1 z} \frac{\partial}{\partial z} E_1(z) + 2ik_2 e^{-2i\omega t + ik_2 z} \frac{\partial}{\partial z} E_2(z) + c.c. \tag{5}$$

In the low efficiency regime, very little light is converted from E_1 to E_2. Therefore, we can assume $E_1(z)$ does not change much from $z = 0$ to $z = L$, and we have $E_1(z) \approx E_1(0)$ and $\partial E_1(z)/\partial z \approx 0$. This further simplify equation (3) to:

$$\nabla^2 E(z,t) - \mu_0\epsilon_0\epsilon_r \frac{\partial^2 E(z,t)}{\partial t^2} = 2ik_2 e^{-2i\omega t + ik_2 z} \frac{\partial}{\partial z} E_2(z) + c.c. = \mu_0 \frac{\partial^2 P_{NL}(z,t)}{\partial t^2}. \tag{6}$$

Question 10:

Plug in the answer from question 8 to equation (6). Show the result.

Question 11:

The result of question 10 is a first order differential equation. Neglect the c.c. part. Can you solve $E_2(z = L)$? (DO NOT plug in any numerical value).
(Hint: $\int e^{ax} dx = e^{ax}/a$).

Question 12:

We'd like to obtain the maximum output of the E_2 at $z = L$ (Use answer from question 11, keep ignoring c.c. part). What are the L that maximize E_2? (plug in numerical value of k_1 and k_2 from question 5).

Question 13:

At what length L will $E_2(z = L)$ become zero? Assume $L > 0$.

Question 14:

Suppose we have a machine to cut silicon waveguide into our designed length L such that we can maximize the second harmonic output. If the cutting machine has a precision of 1 μm (e.g. for a 1 mm waveguide, the final length after the machine cut will be 1mm $\pm$ 1 μm). Is this machine good enough to cut our waveguide?
Fun fact: most optical microscope has resolution around 1 μm. Machine with precision better than 1 μm has to be aligned by other methods.

Question 15:

The intensity of the laser can be expressed as:

$$I = \frac{P}{A} = \frac{nc\epsilon_0}{2}|E|^2, \tag{7}$$

where P is the power of the laser beam, A is the area of cross section of the laser beam.
Let's assume $A = 1$ mm^2, and $\chi_2 = 100$ pm V^{-1}. The power of the input laser beam at 1064 nm is $P_1 = 5$ mW (eye-safe level for near-IR light).
What is the output power of the second harmonic generation at the optimal L given by question 12? What is the nonlinear efficiency Γ?
(Hint: you have to plug in all the values of k_1, k_2 and etc. Also assume $E_1(z)$ and $E_2(z)$ are real number.)

Question 16:

Is the second harmonic light generated from silicon waveguide strong enough as a laser pointer?
Consider a spherical light bulb that emits 1 Watt of light and is 1 meter away from the projector screen. Our laser pointer is the silicon second harmonic generated light with power P_2 given by the previous question. Suppose we are able to focus the laser pointer to a 1cm^2 size spot on the projector screen.
Which light is brighter on the projector screen? The light bulb or the second-harmonic generated light using silicon?
(Hint: compare power per unit area).