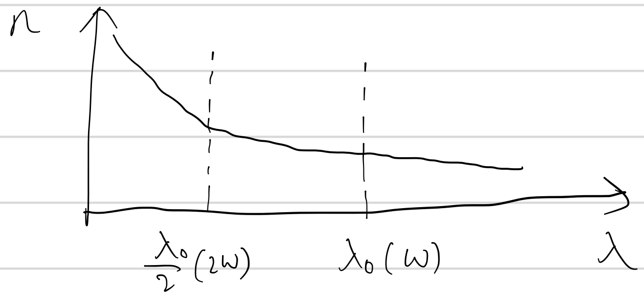
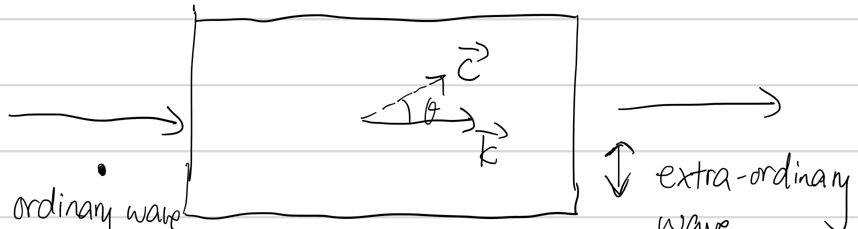
Methods of phase matching

Dispersion. Most of the materials have normal dispersion at visible and near-infred wavelength.



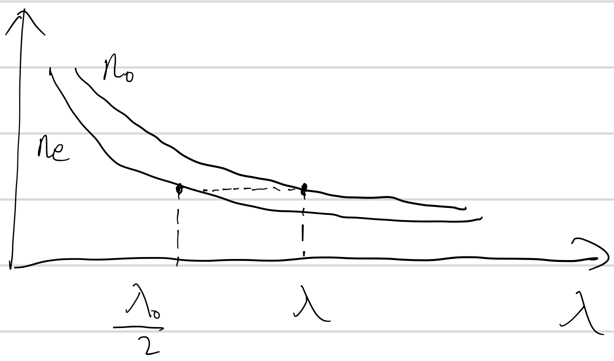
So it's impossible to directly satisfy $n(\omega) = n(2\omega)$

Method I: Birefringent crystal:



(perpendiculary to the plane formed by $\vec{k}$ and optical lattice)

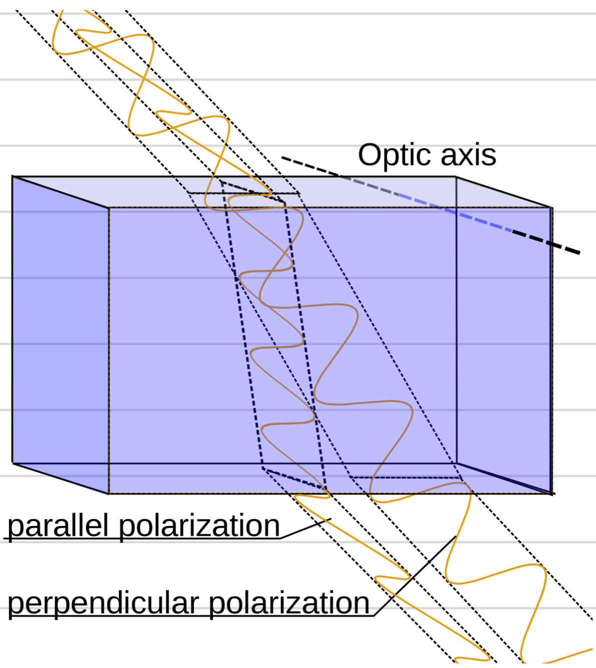
$\vec{c}$ is the optical axis for the crystal material



Possible to phase-match: $n_o(\omega) = n_e(2\omega)$

Question mark: The window seems pretty small!

Note: how to determine optical axis?



From wikipedia

The direction of light is optic axis when no double refraction occurs.

Or symmetry arguement.

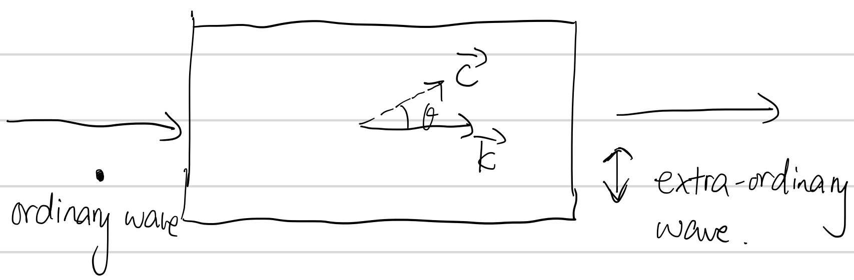
The optical property is symmetric around the optical axis,
When light propagate along the axis, both polarization are perpendicular to the axis, and they have the same refraction index.

How to phase match?

$$\frac{1}{n_e^2(\theta)} = \frac{\sin^2\theta}{\bar{n}_e^2} + \frac{\cos^2\theta}{n_o^2} \; ;$$

$\bar{n}_e$ is the n_e at $\theta = 90°$.
$\updownarrow$ along the $\hat{c}$ axis.

So the extraordinary refractive index can be tuned from n_o ($\theta = 0°$) to $\bar{n}_e$ ($\theta = 90°$)



So rotate the crystal to find the phase matching angle.

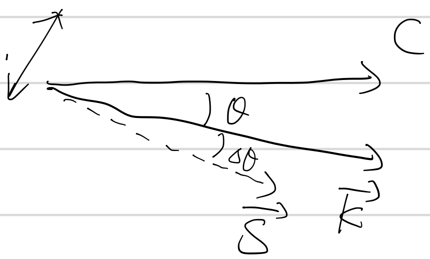
Normally, vendor will mark the optical axis for you.

Note: why the rule of $n_e(\theta)$ is in a square form?

$$\nabla^2 \vec{E} - \frac{\varepsilon_r}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad (\text{linear case}); \quad \varepsilon_r = n_r^2 .$$

This is called "Angle phase match"

Drawback: Direction of phase propagation $\vec{k}$ is different from energy propagation: for extra-ordinary wave.



The angle between $\vec{k}$ and $\vec{S}$ (Poynting vector) is:

$$\Delta\theta = -\frac{1}{n_e}\frac{\partial n_e}{\partial \theta}$$ (milliradian range)

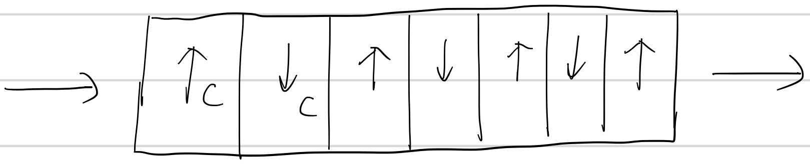
Cause? extra-ordinary wave has components along $\vec{c}$ and perpendicular to $\vec{c}$.

So when calculate $\vec{S} = \vec{E} \times \vec{H}$; not always back to $\vec{k}$ direction.

Limit the interaction range.

Phase-matching can also be slightly tuned by temperature.

Quasi-Phase Matching



periodically flip the optical axis

When you do that: $\chi^{(2)} \to -\chi^{(2)}$ for this propagation direction.

Argument: $\vec{P} = \mu_0 \vec{\vec{\chi}} \vec{E}\vec{E}$;

When flip the axis, polarization change sign. ↑ to ↓ in the frame set-up by optical axis. (along extraordinary wave direction)

So ↑ → ↓

$\vec{P} \to -\vec{P}$.

However, $\vec{E}\vec{E}$ doesn't change sign, because $(-\vec{E})(-\vec{E}) = \vec{E}\vec{E}$.

So $\vec{\vec{\chi}} \to -\vec{\vec{\chi}}$

Take the result from:

$$\frac{\partial E_2(z)}{\partial z} = \frac{2i\mu_0\varepsilon_0\omega^2\chi^{(2)}E_1^2(z=0)}{k_2} e^{i(2k_1-k_2)z}$$

$$E_2(z=L) = \int \frac{2i\mu_0\varepsilon_0\omega^2\chi^{(2)}}{k_2} E_1^2(0)\, e^{i(2k_1-k_2)z} \sim \int i\chi^{(2)} E_1^2 e^{i(2k_1-k_2)z};$$

Now: let

$$\begin{cases} \chi^{(2)} = d, & \text{when } 2N\pi < (2k_1-k_2)z < (2N+1)\pi \\ \chi^{(2)} = -d, & \text{when } (2N+1)\pi < (2k_1-k_2)z < (2N+2)\pi. \end{cases}$$

Why? Because previously, the intergral from $0 \to \pi$ was canceled by $\pi \to 2\pi$.

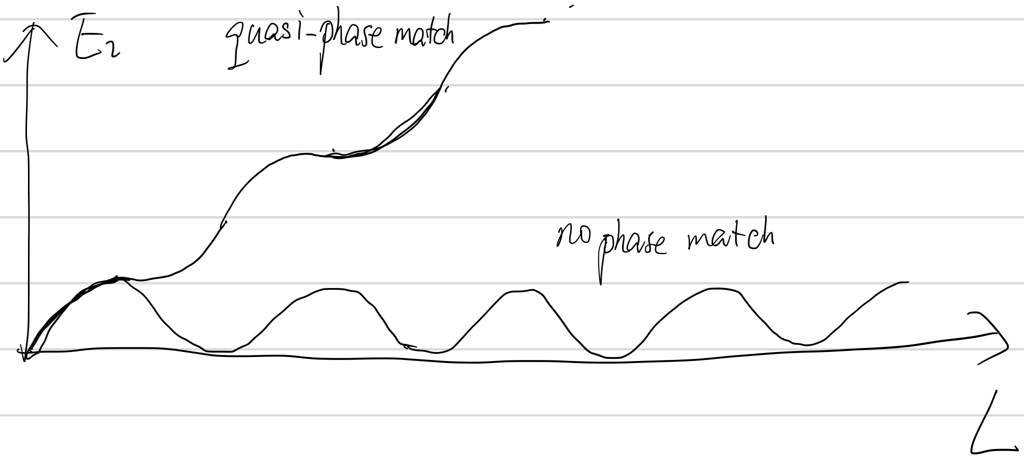
So if flip the sign of $\pi \to 2\pi$, we get enhanced result, instead of cancellation.

at $2N\pi < (2k_1-k_2)z < (2N+1)\pi$.

$$\int_{\frac{2N\pi}{2k_1-k_2}}^{\frac{(2N+1)\pi}{2k_1-k_2}} i\chi^{(2)} E_1^2 e^{i(2k_1-k_2)z}\, dz = \frac{d\cdot E_1^2}{(2k_1-k_2)}\left(e^{i\pi}-e^{i0}\right) = -\frac{2d\cdot E_1^2}{2k_1-k_2}$$

$$\int_{\frac{(2N+1)\pi}{2k_1-k_2}}^{\frac{(2N+2)\pi}{2k_1-k_2}} i\chi^{(2)} E_1^2 e^{i(2k_1-k_2)z}\, dz = \frac{-dE_1^2}{2k_1-k_2}\left(e^{i2\pi}-e^{i\pi}\right) = \frac{-2dE_1^2}{2k_1-k_2}$$

So:



Fabrication method:
use high static voltage to invert the orientation of ferroelectrical domain, and therefore $\vec{c}$ axis.

Voltage around 20 kV/mm.

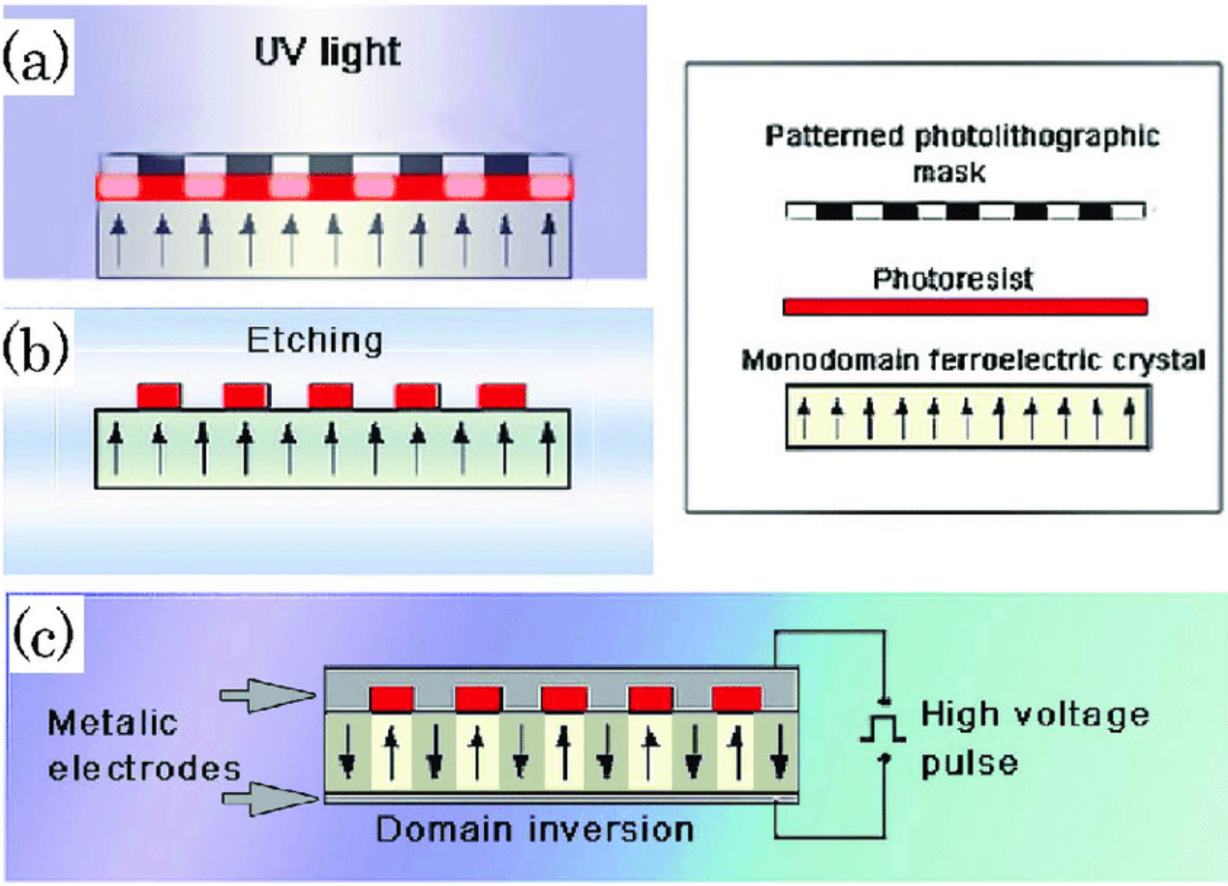


Figure source: Linear electrooptic microscopy : applications to micro and nano-structured materials